MATH 54 − FINAL EXAM − STUDY GUIDE

PEYAM RYAN TABRIZIAN

Note: The Final Exam is on **Friday, August 10th** in 4 Evans from 12:05 pm to 2 pm. It covers sections $6.1 - 6.5$ and 6.7 of the Linear Algebra book and sections $10.2 - 10.6$ of the Differential Equations book.

Note: 1.3.4 means 'Problem 4 in section 1.3'

Chapter 6: Inner Products and Norms

- Determine if a set is orthogonal, or orthonormal (6.2.3, 6.2.21), and if a set is only orthogonal, know how to normalize it (try it out on 6.2.3)
- Use orthogonality to find a, b, c such that $\mathbf{x} = a\mathbf{u} + b\mathbf{v} + c\mathbf{w}$ (6.2.7, 6.2.9) and remember the 'hugging'-analogy/demo
- Find the orthogonal projection $\hat{\mathbf{x}}$ of $\mathbf{x}$ on a subspace W or a line L (6.2.11, 6.3.5) And use this to:
  - Find a vector orthogonal to W
  - Write $\mathbf{x}$ as the sum of two vectors, one in W and another one orthogonal to W (6.3.1, 6.3.7)
  - Find the smallest distance between $\mathbf{x}$ and W (6.2.15, 6.3.11)
- Determine if a matrix Q is orthogonal, and use this to calculate $Q^T Q$, Q^{-1}, $\|Q\mathbf{x}\|$ (6.2.29)
- Know what QQ^T means in terms of orthogonal projections (6.3.17)
- Use the Gram-Schmidt process to produce an orthogonal or orthonormal basis of a subspace W spanned by some vectors (6.4.1, 6.4.3, 6.4.5, 6.4.7, 6.4.9, 6.4.11)
- Find the least-squares solution (and least-squares error) of an inconsistent system of equations (6.5.1, 6.5.3, 6.5.5, 6.5.9, 6.5.11)
- Also understand *why* least-squares work, in terms of orthogonal projection (see lecture on least-squares, and 6.5.9)
- Know when an equation has a unique least-squares solution (it's when the columns of A are linearly independent)
  Note: There will be no question on LU or QR factorizations
  Note: There will be no questions on linear models
- Find inner products, lengths, and orthogonal projections of functions f and g using $f \cdot g = \int_a^b f(t)g(t)dt$ (6.7.21, 6.7.23)
- Use the Gram-Schmidt process to find an orthonormal basis of **functions** (6.7.25, 6.7.26)
- Remember the Cauchy-Schwarz inequality (6.7.19, 6.7.20)
- Know how to do cute mini-proofs with dot products (what I mean is look at 6.1.24, 6.7.17, 6.7.18)

Date: Friday, August 10th, 2012.

CHAPTER 10: PARTIAL DIFFERENTIAL EQUATIONS

- Solve boundary-value problems (10.2.1, 10.2.3, 10.2.4)
- Find all the numbers λ such that $y'' + \lambda y$ has a nonzero solution (10.2.9, 10.2.10)
- Use separation of variables to solve a PDE (see heat and wave equations, and also 10.2.27)
- Calculate the Fourier series of a function f on a given interval, and determine to which function that Fourier series converges (10.3.9, 10.3.12, 10.3.13, 10.3.17, 10.3.20, 10.3.21)
- Calculate the Fourier cosine/sine series for a function f, and determine to which function that Fourier series converges to (10.4.5, 10.4.7, 10.4.11, 10.4.13, for the second part, you need to understand oddification and evenification, so see 10.4.1, 10.4.3)
- Use separation of variables and Fourier series to solve the heat and wave equations subject to various boundary/initial conditions (10.5.1, 10.5.2, 10.5.3, 10.5.5, 10.5.7, 10.6.1, 10.6.2, 10.6.3)
  Note: There will be no question on the Laplace equation, nor will there be any questions on inhomogeneous heat/wave equations
  Note: There will be no question on complex exponential Fourier series

TRUE/FALSE EXTRAVAGANZA

Check out the following set of T/F questions (solutions are in the HW hints, but beware, there might be mistakes, e-mail me whenever something seems to be wrong): 6.1.19(abe), 6.1.20(abcd), 6.2.23, 6.2.24, 6.3.21, 6.3.22(acde), 6.4.17(ab), 6.4.18(ab), 6.5.17, 6.5.18(abcd)

Also, review the two T/F extravaganzas that we covered in lecture (on July $25/26$), those are very good/important!

Note: There will be **NO** T/F questions about differential equations, and there will be **NO** T/F questions with justifications. However, there will be **5** T/F questions without justifications. They will *all* be linear algebra questions!

CONCEPTS

Here are a couple of concepts we learned so far. You **don't** have to memorize the definitions, just have a rough idea of what those things are

- Dot (Inner) product, Norm
- Orthogonal, Orthonormal
- Orthogonal matrix
- Orthogonal projection
- Gram-Schmidt process (how it works)
- Least-Squares (why it works), Least-squares error
- Cauchy-Schwarz inequality
- $f \cdot g$
- Separation of variables
- Eigenvalue/Eigenfunction
- (full) Fourier series, Fourier cosine/sine series, evenification/oddification